PROBLEM SET 4: APPLICATIONS OF THE INTEGRAL — Due: October 1, 2018
MATH 141, FC02 — Instructor: Maxx Cho

Name: _____

You must show your work and demonstrate your understanding to receive full credit. **Where appropriate, a picture must accompany your integral to justify its set-up.**

1. Complete the following centroid problems:
   (1) (2 points) Two children weighing 18 and 23 kilograms are sitting on opposite sides of a see-saw, each 2 meters from the axis of rotation. Where on the see-saw should a third child weighing 10 kilograms sit in order to achieve equilibrium?
   (2) (4 points) Let R be the region between the graphs of $f(x) = (x+3)^2$ and $g(x) = (x-3)^2$. Find the moments M_x and M_y, then find the centroid $(\overline{x}, \overline{y})$.
   (3) (2 points) Find the centroid of a hexagon with vertices at $(2,0), (3,1), (3,5), (2,6), (1,5)$, and $(1,1)$.
   (4) (4 points) A region R consists of a square bounded by the lines $x = -8, x = 8, y = 0, y = -16$, and a half-disk bounded by the semicircle $y = \sqrt{64 - x^2}$ and the line $y = 0$. Find the centroid of this region.

2. Complete the following work problems:
   (1) (4 points) When a certain spring is expanded 7 centimeters from its natural position, the force necessary to hold it in that position is 1400 dynes. Find the work required to stretch the spring an additional 7 centimeters.
   (2) (4 points) A swimming pool full of water has the shape of a right circular cylinder with radius 16 feet and height 10 feet. Find the work required to pump all the water to a platform 2 feet above the pool.
   (3) (4 points) A water tank has the shape of an inverted circular cone with height 4 feet and radius 4 feet. The tank is full of water. Find the work required to pump all the water out of the tank until only 2 feet of water remains at the bottom.

3. Consider the curve C given by the parametric equations $x(t) = t$ and $y(t) = (t-1)^2$ on $0 \leq t \leq 1$. Let R be the region formed by C, the x-axis, and the y-axis. Revolve R around the y-axis to form the solid D.
   (1) (3 points) Find the volume of D using the theorem of Pappus and Guldin.
   (2) (3 points) Find the volume of the same solid D using cross sections.

4. (5 points) Consider the curve C parametrized by $x(t) = -1 + 9\sin(-8t)$ and $y = \frac{1}{2} - 9\cos(-8t)$ for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.
   (1) What is the radius of the circle? Where is the center of the circle?
   (2) Is the orientation of the circle clockwise or counter-clockwise? How many times does the parametrization traverse the circle?
   (3) What is the arc length on the domain of parametrization? (Hint: You can get the answer without an integral.)

5. Find the arc length of the following curves:
   (1) (5 points) The curve is parametrized by $x(t) = e^t \sin t$ and $y(t) = e^t \cos t$ on $0 \leq t \leq \pi$.
   (2) (5 points) The curve is the graph of the function $f(x) = x^4 + \frac{1}{32x^2}$ for $1 \leq x \leq 2$.

6. (2 points) Do the following integral:

$$\int_0^{\pi} \sqrt{\cos^2 x}\, dx$$

Why did I ask this question?

7. (3 points) The base of a solid is circle with radius 2. The cross-sections perpendicular to a fixed diameter of the base are equilateral triangles. Find the volume of the solid.

8. (Extra Credit 5 points) Suppose there is a cylinder of cheese with radius a. A wedge is cut out of the cylinder by first making a cut halfway through the cheese perpendicular to the axis of the cylinder and then making another cut halfway through the cheese at an angle θ with respect to the first cut. Find the volume of the wedge of cheese removed. (Hint: the cross sections of the cheese wedge perpendicular to the first cut are rectangles.)